CONTRACEPTION STRATEGIES AND FERTILITY OUTCOMES: ESTIMATES OF A STOCHASTIC MODEL OF REPRODUCTION

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In this paper we report some results from an ongoing study in the economics of fertility. Space limitations preclude a complete summary of our work. Accordingly, in this paper we briefly sketch the economic model and discuss how it differs from previous work in economic demography. We then discuss a statistical problem that arises in estimating the model and illustrate the practical importance of this issue using data from the 1965 Princeton National Fertility Study. For a more complete description of our work, readers are referred to Heckman and Willis, 1974.

I
The Model

Previous work in economic demography assumes that families choose a desired number of children and a desired amount of child quality in a world of perfect foresight. This approach, developed most fully in Willis (1973), neglects the uncertainty inherent in the fertility process. In this paper a family is viewed as controlling parameters of a stochastic birth process through choice of contraceptive techniques and levels of use effectiveness. There are monetary, time, and psychic costs associated with each form of contraception and level of use effectiveness. Families are assumed to maximize expected utility over a finite horizon, and contraception decisions are made with this view in mind.

A family's decision problem may formally be represented as a dynamic programming problem. Except in simple cases, a complete analytical solution to the problem is unavailable. Nonetheless certain insights do emerge from the analysis: (1) the fertility strategy of a family, and its outcome, depend on its previous history of fertility outcomes, including realized spacing intervals. (2) The stochastic models of reproduction advanced by Perrin and Sheps (1964) are embedded in a choice theoretic framework. (3) Given sufficient data on the time series of a family's income and wage rates, testable hypotheses may be generated about the life cycle history of contraceptive choice and fertility outcomes. (4) Timing, spacing, and final number of children are all generated by a common probability process that can be altered by contraceptive decisions.

II
A Statistical Problem

The model of the previous section immediately suggests that the family's fertility decisions may be represented by a decision tree. Conditional on a sequence of realized events, families make fertility decisions. The probabilities in the observed stochastic birth process may be parameterized, and hypotheses about the effect of such economic variables as the education of the wife, her age at marriage, and the husband's income may be tested by determining at what stages, and in which decisions, economic variables can be said to contribute anything to explaining fertility outcomes. Even in the absence of a fully developed theory of family planning, estimates of the constituent probabilities allow us to account for the importance of economic variables in the various components of the birth process.

In estimating these probabilities from a random sample of individuals, it is important to note that unless very strong statistical assumptions are made, the simple semi-Markov probability structure does not lead to a simple likelihood function in which estimated parameterized probabilities can accurately be said to predict the probabilities of observed events for individuals. To see that this is so, it is important to distinguish three sources of variation in observed birth intervals among individuals: (1) purely random factors that arise independently in each time period, and are independent of random factors in other time periods, (2) random factors, including unobservable variables, that are correlated across time periods, (3) deterministic variables such as income and education that can be measured, and which are assumed to affect the probabilities.

To fix ideas, suppose we are concerned solely with estimating the parameters of the probability process determining whether a woman has a first pregnancy. Inherent in the model is the notion of a time series of events. A woman has a first pregnancy in month j only if she has not had a first pregnancy in months $1,\dots,j-1$. The most general way to model this probability is to imagine a set of continuous random variables S_1, $S_2,\dots$, which may be thought of as index functions. The S_i, $i=1,\dots,\infty$, are assumed to be intercorrelated. The event of a woman becoming pregnant in the first interval depends on what value the "wheel of chance" throws up for S_i. Suppose that her education E is the only economic variable of interest. We may then define $\alpha_o+\alpha_1E$ so that if $S_1<\alpha_o+\alpha_1E$ a woman becomes pregnant in the first interval and leaves the sample while if the inequality is reversed, the woman is not pregnant and stays in the sample. The probability of a woman becoming pregnant in the jth interval is thus

$$\text{(1)}\quad Pr(S_1> \alpha_o+\alpha_1E,\dots,S_{j-1}>\alpha_o+\alpha_1E_1,S_j<\alpha_o+\alpha_1E) \ .$$

If we assume that the S_i are independently and identically distributed, this probability may be written as

$$\text{(2)}\quad \prod_{i=1}^{j-1} Pr(S_i>\alpha_o+\alpha_1E)\ Pr(S_j<\alpha_o+\alpha_1E) \ .$$

If each S_i is assumed to be distributed normally with mean zero, and variance σ_S^2, the probability statement may be written using probit functions as

$$(3)\quad \left[\int_{\frac{\alpha_o+\alpha_1 E}{\sigma_s}}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}dt\right]^{j-1} \int_{-\infty}^{\frac{\alpha_o+\alpha_1 E}{\sigma_s}} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}dt$$

If the S_i were assumed to be logistically distributed, a similar probability statement using cumulative logistics could easily be written.

If the S_i for all women are generated by the same random process, we may use the principle of maximum likelihood to estimate $\frac{\alpha_o}{\sigma_s}$ and $\frac{\alpha_1}{\sigma_s}$ by taking a sample of women with different birth intervals, and choosing parameter values which maximize the probability of observing the sample distribution of birth intervals.

Note, however, a crucial step in the argument. We assumed that over time, the S_i were independently distributed. This assumption rules out serial correlation in the S sequence. Such serial correlation may naturally arise if there are unmeasured random variables which remain at, or near the same level, over time for a given individual, but which are randomly distributed among individuals. For example, unmeasured components of fecundability (e.g. semen counts of husbands, tastes for coital activity, and variations in contraceptive efficiency) plausibly have a persistent component for the same individual across time periods although these components may vary widely among individuals.[1] Similarly, important economic variables may be missing in a given body of data.[2]

Following a convention in the analysis of covariance, we may decompose S_i into two components

$$(4)\quad S_i = U_i + \varepsilon$$

where U_i is a random variable with mean zero and variance σ_u^2, and ε is a random variable with mean zero, and variance σ_ε^2. We further assume, letting "E" be the mathematical expectation, that

$$(5)\quad E(U_iU_j) = 0, \quad i \neq j$$

$$E(U_i\varepsilon) = 0 \quad .$$

Then S_i is a random variable with mean

$$E(S_i) = 0$$

and

$$(6)\quad \begin{aligned} E(S_iS_j) &= \sigma_\varepsilon^2 \quad i \neq j \\ &= \sigma_\varepsilon^2+\sigma_j^2, \quad i=j \end{aligned} \quad ,$$

so that the correlation coefficient between S_i in any two periods, ρ, may be defined as

$$(7)\quad \rho = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} \quad .$$

Clearly, it is possible to imagine more general intercorrelation relationships such as a first-order Markov process. These generalizations are straightforward and, since they are not of direct interest in this paper, are not pursued here. If intercorrelation applies because there are persistent omitted variables, the probability of a woman becoming pregnant in interval j can no longer be written in the simple form of equation (2) (or if S is assumed normal, as in equation (3)). To see what the appropriate probability statement becomes, note that in general we may write the probability of the event conditional on a given value of ε as

$$(8)\quad Pr(S_1>\alpha_o+\alpha_1E,\ldots,S_{j-1}>\alpha_o+\alpha_1E,S_j<\alpha_o+\alpha_1E/\varepsilon) \ .$$

But note that if ε is held fixed, the distribution of S_i conditional on $\varepsilon=\tilde{\varepsilon}$ must satisfy the following properties

$$(9)\quad \begin{aligned} E(S_i/\tilde{\varepsilon}) &= \tilde{\varepsilon} \\ E(S_iS_j/\tilde{\varepsilon}) &= \tilde{\varepsilon}^2 \quad i\neq j \\ &= \sigma_u^2 + \tilde{\varepsilon}^2 \end{aligned}$$

and since the U_i are independent the conditional values of S_i are also independent.

Then we see that

$$(10)\quad \begin{aligned} &Pr(S_1>\alpha_o+\alpha_1E,\ldots,S_{i-1}>\alpha_o+\alpha_1E,S_j<\alpha_o+\alpha_1E/\tilde{\varepsilon}) \\ &= Pr(S_1>\alpha_o+\alpha_1E/\tilde{\varepsilon})Pr(S_2>\alpha_o+\alpha_1E/\tilde{\varepsilon})\ldots \\ &\qquad Pr(S_j<\alpha_o+\alpha_1E/\tilde{\varepsilon}) \end{aligned}$$

so that conditional on $\varepsilon=\tilde{\varepsilon}$ we reach precisely the same functional form as in equation (2) where persistent omitted variables are ignored. However, to solve back to the probability of interest, where ε is permitted to vary between plus and minus infinity, we note that the <u>unconditional</u> probability may be written as

$$(11)\quad \int_{-\infty}^{\infty} Pr(S_1>\alpha_o+\alpha_1E/\varepsilon)Pr(S_2>\alpha_o+\alpha_1E/\varepsilon)\ldots\ldots Pr(S_j<\alpha_o+\alpha_1E/\varepsilon)h(\varepsilon)d\varepsilon$$

where $h(\varepsilon)$ is the marginal density function of ε, and ε is permitted to vary over all possible values, as before.

In the special case with S normally distributed with zero mean and variance $\sigma_\varepsilon^2+\sigma_u^2$, equation (11) is seen to be the integral of a multivariate, normal density with equicorrelated variates, and common correlation coefficient ρ. The expression adopted for the computations is

$$(12)\quad \int_{-\infty}^{\infty}\left[\left[\int_{\frac{\alpha_o^*+\alpha_1^*E+\rho^{1/2}q}{(1-\rho)^{1/2}}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt\right]^{j-1}\int_{-\infty}^{\frac{\alpha_o^*+\alpha_1^*E+\rho^{1/2}q}{(1-\rho)^{1/2}}}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt\right]\frac{1}{\sqrt{2\pi}}e^{-q^2/2}dq$$

where $\alpha_o^* = \frac{\alpha_o}{(\sigma_u^2+\sigma_\varepsilon^2)^{1/2}}$ and $\alpha_1^* = \frac{\alpha_1}{(\sigma_u^2+\sigma_\varepsilon^2)^{1/2}}$.

If no serial correlation is present (ρ=0), this expression collapses to equation (3). In the more general case, ρ allows us to measure the proportion of total variance in the index S explained by systematic correlated components.

Notice that there is an alternative "incidental parameters" argument that leads directly to equations (11) and (12). Suppose it is argued that in an ordinary probit model a disturbance "ε" appears. This may be viewed as an incidental parameter with density function h(ε). Following a suggestion of Kiefer and Wolfowitz (1956), the problem of incidental parameters has precisely the general solution written in equation (11) or the specific solution for the normal case as in equation (12).

Yet another interpretation of these results is possible. An individual may be modeled as having a geometric probability process characterizing the probabilities of pregnancy at each interval for a given value of ε. "ε" is, in fact, a random variable governed by a density function h(ε). Then the true probability of pregnancy at month j is a continuous mixture of geometric processes and is given by equation (11).

Elsewhere we demonstrate that estimates of the population probabilities that neglect the serial correlation phenomenon impose the constraint that the conditional probability of pregnancy in a given month for a group of women with identical economic characteristics is the same for all months from the onset of marriage. If persistent omitted variables are present (i.e. ρ in equation (12) is nonzero), it can be shown that this conditional probability declines with the length of the interval since marriage because more fecund women tend to become pregnant and hence drop out of the sample eligible for a first birth. If ρ is inappropriately assumed to be zero, estimates of the effect of economic variables on the probability of conception are biased.

III
Empirical Results

This section presents estimates of the monthly probability of conception in the first pregnancy interval following marriage using the econometric model discussed in the preceding section. The data consist of a sample of white non-Catholic women, married once with husband present for 15-19 years from the 1965 Princeton National Fertility Study.[3] The sample of all such women was reduced by eliminating women who reported premarital conceptions or who had missing values for relevant variables. The sample was then divided into two groups, contraceptors and noncontraceptors, on the basis of the woman's response to a question concerning the contraceptive methods she used before her first pregnancy (or in her current interval if she had not had a pregnancy). Three variables (wife's education (W), wife's age (A), and husband's predicted income at age 40 (H)) are expected to influence the monthly probability of conception.

Women, in each subsample were "followed" for a maximum of 120 months beginning with their first month of marriage. Among the noncontraceptors we estimate the monthly probability of conception in the first pregnancy interval by maximizing the appropriate likelihood function.[4] That is, using the functional form of the likelihood function implied by (12) we estimate parameters which maximize the likelihood of observing the events that occurred in this subsample. These events are (1) that a given woman conceived in month j (j=1, ..., 120) or (2) that she went 120 months without conceiving. Among the contraceptors, we estimate in similar fashion the monthly probability of conception given that the woman is contracepting. In this case, the events we observe are (1) that a woman conceives in month j while using a contraceptive; (2) that the woman uses a contraceptive for k months without conceiving, at which time she discontinues contraception (this decision is treated as an exogenous event); or (3) she continues using contraception for 120 months and does not conceive.[5]

Parameter estimates for the noncontraceptors are presented in Table 1A and for contraceptors in Table 1B. In each group, we estimated six models which differ in the number of parameters estimated in order to determine the statistical significance of individual parameters or sets of parameters using likelihood ratio tests.

Among these parameters, we have a particular interest in the magnitude of the serial correlation coefficient, ρ, its statistical significance and the influence of its inclusion or exclusion from the econometric model on the other parameters of the model (i.e., the constant term, α_0, and the coefficients of A, W, and H which are, respectively, α_1, α_2 and α_3). Accordingly, we present two estimates of each set of α's in Table 1, one in which ρ is constrained to be zero and one in which ρ is free to assume a nonzero value. The sign of a coefficient indicates the direction of effect of the variable on the probability of <u>not</u> conceiving in a given month. For example, in Table 1 the positive coefficient for α_1 suggests that the later the wife's age at marriage, the less likely she is to conceive in a given month.

In Table 1 ρ is positive and statistically significant in every instance. Among the noncontraceptors ρ=0.450 when only the constant term is entered and falls to 0.426 when the wife's age at marriage is held constant, but does not fall any further when wife's education and husband's predicted income are added to the model. Similarly, the estimate of ρ in the contracepting subsample falls from 0.549 to 0.531 when A is held constant and to 0.526 when W and H are also held constant. If we recall that the definition of ρ is the fraction of persistent variance (σ_ε^2) in total variance ($\sigma_\varepsilon^2+\sigma_u^2$), the decrease in ρ is easily understood as showing that the exogenous variable A in the noncontracepting subsample and the variables A, H and W in the contracepting subsample contribute to the persistent component of variation in conception probabilities among women in the two subsamples. The small size of the decrease in ρ, however, also shows that the contribution of other factors we have not held constant constitutes the major fraction of persistent variation. This suggests that it is unlikely that the heterogeneity problem can be overcome simply by holding constant a number of observable variables.

The size of the decrease in ρ caused by the addition of exogenous variables is, of course, related to the statistical significance of these variables. The wife's age at marriage is the only variable to pass a test of statistical significance at conventional levels in either subsample.

Estimates of the monthly probability of conception and the effects of changes in exogenous variables on that probability differ substantially depending on whether or not serial correlation is taken into account. In Table 2, we present examples of estimates of levels in the monthly probability of conception among noncontraceptors and contraceptors with and without ρ constrained to equal zero. To make the contrasts, we present estimates of the probabilities of conception in the first month after marriage. These estimates are derived from Table 1.

For example, line (1') in Table 1 maps into the second column of line A.1 in Table 2. Similarly, line (1) in Table 1 maps into the first column of line A.1. It is easy to see that the bias from not allowing for serial correlation is quite large. For contraceptors (line A.2 in Table 2) a similar result holds. In both cases, the monthly probability of conception is seriously understated when ρ is constrained to be zero.

In Table 2B, we evaluate the monthly probability of conception for one value of wife's age at marriage for noncontraceptors and contraceptors with and without ρ constrained to be zero from parameter estimates in lines (2), (2'), (5) and (5') in Table 1. Here, we notice that the bias arising from estimates neglecting serial correlation is large.

Similarly, dramatically different estimates for the effect of economic variables on the probability of conception result when allowance is made for serial correlation.

TABLE 1

ESTIMATES OF PARAMETERS OF MODEL FOR CONTRACEPTORS AND NONCONTRACEPTORS IN FIRST PREGNANCY INTERVAL AFTER MARRIAGE

	Constant (α_o)	ρ	Wife's Age at Marriage (α_1)	Wife's Education (α_2)	Husband's* Predicted Income (α_3)	Log e Likelihood
A. Noncontraceptors (177 Observations)						
(1)	2.016					-692.71
(1')	1.214	0.450				-619.50
(2)	1.154		0.0033			-680.42
(2')	0.172	0.426	0.0042			-613.36
(3)	1.022		0.0031	0.017	-0.0033	-679.80
(3')	0.132	0.426	0.0041	-0.004	0.0125	-613.33
B. Contraceptors (246 Observations)						
(4)	2.264					-336.92
(4')	1.780	0.549				-319.43
(5)	1.307		0.0038			-332.42
(5')	0.646	0.531	0.0046			-316.32
(6)	1.072		0.0036	-0.0016	0.0387	-331.82
(6')	0.943	0.526	0.0042	-0.0068	0.0903	-314.89

*This variable is estimated from a regression of husband's income on his education and age, and arbitrarily assigning the value of 40 for age so that the regression prediction is an estimate of husband's permanent income.

TABLE 2

ESTIMATES OF MONTHLY PROBABILITY OF CONCEPTION DERIVED FROM PARAMETER ESTIMATES IN TABLE 1

	(a) Serial Correlation Ignored ($\rho=0$)	(b) Serial Correlation Allowed ($\rho>0$)
A. Model with Constant Term Only (α_0)		
1. Noncontraceptors	.022	.113
2. Contraceptors	.012	.038
B. Effect of Wife's Age at Marriage (Model with α_0, α_1)		
Noncontraceptors		
3. Age 20	.026	.122
Contraceptors		
4. Age 20	.013	.040

Footnotes

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[1]The problem of heterogeneity is considered in a demographic context by Sheps (1964), Potter and Parker (1964) and Sheps and Menken (1972).

[2]In this paper, we abstract from the further problem that the unobserved components may be correlated with the included variables.

[3]The 1965 National Fertility Study, conducted by Norman B. Ryder and Charles F. Westoff, is a cross-section national probability sample of 5,617 U.S. married women which is described in detail in Ryder and Westoff (1971). For our purposes, its most important characteristics are that it records (retrospectively) the date of marriage of the woman, the dates of each pregnancy termination, the use of contraception in each pregnancy interval, and the time of discontinuation of contraception prior to pregnancy in addition to a number of household characteristics such as income and education.

[4]The methods used are described in Goldfeld and Quandt (1972, Ch. 1). Two algorithms, Powell and GRADX, were used in tandem to ensure that the estimates are stable. That is, in the first stage the parameters of the likelihood function were estimated by the Powell method. These parameters were then given as initial values in a GRADX optimization procedure whose final parameter values are reported in this paper. The computer program, written by C. Ates Dagli and Ralph Shnelvar is available from the authors on request.

[5]Our data only record whether a woman contracepted in a given pregnancy interval and when and if she discontinued contraception. They do not record when she began contracepting or any other interruptions in contraceptions other than the final decision to discontinue.

References

Goldfeld, S.M. and R.E. Quandt, Nonlinear Methods in Econometrics, Amsterdam-London, North-Holland, 1972.

Heckman, J. and R. Willis, "Estimates of a Stochastic Model of Reproduction: An Econometric Approach," in Household Production and Consumption, ed. by N. Terleckyj, Studies in Income and Wealth, Vol. 139, 1974.

Kendall, Maurice and Stuart, Alan, The Advanced Theory of Statistics, Vol. I, New York, Hafner Publishing, 1969.

Kiefer, J. and Wolfowitz, J. "Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters," Annals of Mathematical Statistics, Vol. 27, No. 4, December, 1956.

Pearson, J., "Contributions to the Mathematical Theory of Evolution. Disection of Frequency Curves," Philosophical Transactions of the Royal Society, Series A, Vol. 185, 1894.

Perrin, Edward and Mindell C. Sheps, "Human Reproduction: A Stochastic Process," Biometrics, Vol. 20, March, 1964, pp. 28-45.

Quandt, R., "New Methods for Estimating Switching Regressions," Journal of the American Statistical Association, June, 1972.

Sheps, M.C., "On the Time Required for Contraception," Population Studies 18, 1964, pp. 85-97.

Sheps, M.C. and J. Menken, "On Estimating the Risk of Conception from Censored Data," T.N.E. Greville, Population Dynamics, Academic Press, New York, 1972, pp. 167-200.

Willis, Robert J., "A New Approach to the Economic Theory of Fertility Behavior," Journal of Political Economy, Vol. 81, No. 2, Part II, March/April, 1973, pp. s14-s64.

Zellner, A., "Bayesian and Non-Bayesian Analysis of the Regression Model with Multivariate Student-t Error Terms, May, 1973 (unpublished paper).

Ryder, Norman B., "Contraceptive Failure in the United States," Family Planning Perspectives, Vol. 5, No. 3, Summer 1973, pp. 133-44.

Ryder, Norman B. and Charles F. Westoff, Reproduction in the United States 1965, Princeton: Princeton University Press, 1971.